# Investments Answers to Problem Set 2 Lent 2022

**True-False (plus reasons; I am expecting no more than two-three sentences for each):**

1. A share that has a weekly volatility of returns of 5% will have an annual volatility of returns of about 35%.

**True. To annualise weekly returns, multiply by the square root of the number of weeks in a year. With the square root of 52 being 7.2, the annual volatility is 7.2 x 5% = 36%.**

1. If share *A* is much more volatile than share *B*, then in the long term it is likely to do better than share *B*.

**False. There is no empirical evidence, and no theoretical reason, for believing that volatility by itself attracts a reward. While investors may require reward for holding volatile *portfolios*, the volatility of an individual share may make very little difference to the volatility of the portfolio as a whole if its risk is uncorrelated with the risk of the rest of the portfolio.**

1. If share *A* and share *B* have a correlation of returns of 0.8, and if share *B* is very volatile then share *A*  must also be quite volatile.

**False. Even if the two shares were perfectly correlated, they could have very different volatilities. For example if share B is like a portfolio that is 10% share A and 90% cash, it will be perfectly correlated with A (assuming the cash is non-volatile) but it will have only 10% of the volatility.**

1. If two shares have zero correlation, then it is possible to construct a portfolio consisting of the two shares that has zero volatility.

**False. If the two shares have volatility a and b respectively then a portfolio that has proportion x in a and 1-x in b has variance x2a2 + (1-x)2b2 which can only be zero if a or b is zero.**

1. If two portfolios are both on the mean-variance efficient frontier, then the one that has the higher expected return will also have the higher volatility.

**True. If they are both mean-variance efficient, then neither can be dominated by the other. (If you interpret the frontier to include the entire frontier region, including the frontier below the feasible set, then the answer is false).**

1. Assuming the CAPM is correct, a share with a higher beta will have a higher expected return than one with a lower beta even if it has much lower total risk.

**True. The CAPM says that expected return is proportional to beta. And a firm with higher beta can have lower total risk if it has much smaller idiosyncratic risk.**

**Questions**

1. If a share has a annual volatility of 25%, and a correlation with the market of 0.3, and the market has an annual volatility of 15%, what is the beta of the share, and what is its idiosyncratic risk?



**The market risk of the share is therefore 0.5x15% or 7.5%. The rest of the risk of the share is idiosyncratic. Total risk2 = market risk2 + idiosyncratic risk2, so idiosyncratic risk2 = 25%2 – 7.5%2 = 0.056875, and idiosyncratic risk = 23.85%.**

1. In the previous question, if the risk-free rate is 4% and the market risk premium (the difference between the expected return on the market and the risk-free rate) is 6%, what is the expected return on the share?



1. If two shares each have a beta of 1 and annual volatility of 30% while the market has an annual volatility of 20%, what is the correlation between each of the shares and the market? What is the covariance of returns of the two shares, and what is the correlation between the two shares, assuming that the idiosyncratic risk of the two shares is uncorrelated?

**Call the shares a and b. For the first question, look at share a (b is identical):**



**The return on each share can be written as:**



**Where the ’s are constant, *rM* is the market return and the ‘s are the idiosyncratic return. The covariance can be written as:**



**Using the definition of correlation:**



1. How many shares would you need to hold in a portfolio to have an annual volatility of no more than 22%, assuming that they are as in the previous question (volatility of 30%, beta of 1 and idiosyncratic risk uncorrelated across stocks)?

**Numbering the shares 1 to *N*, the variance of a portfolio composed of equal holdings of the *N* shares is:**



**When *i = j* ij = 0.32 = 0.09. Otherwise, ij = 0.04 (from above). So:**



**If we want p < 22%, then we need:**



**At least six shares are needed.**

The pension fund …

* 1. Assuming that the trustees want to remain fully invested in equities, draw the set of feasible portfolios (that is all possible combinations of mean and standard deviation) if the fund is invested in a mix of US and Japanese equities for both forecasts. Is a 100% US portfolio efficient? If it is not efficient, identify a better portfolio that is efficient.

**On the optimistic forecast, both markets have the same mean, so diversifying into Japan would have no effect on the mean return. If a proportion** *a* **is put into the Japanese market the volatility would be:**



**The term inside the square root sign is 0.03784***a***2 – 0.03104***a* **+ 0.0256. Differentiating with respect to** *a* **this gives 0.07568***a* **– 0.03104. The volatility is minimized by setting the derivative equal to zero, so *a* = 3104/7568 = 0.41. So to get minimum risk, it is necessary to put 41% of the money into Japan. This is the efficient portfolio. It has a volatility of 13.9%.**

**If the expected return on Japan is only 6%, then there is a trade-off between risk and return shown below in red (the crosses represent 100% US or 100% Japan, and the minimum variance portfolios; the blue line is for the optimistic case, the dotted lines allow for going short):**



**The 100% US portfolio is efficient if you want to maximise return.**

* 1. The US interest rate is 3%. The fund is free to put some of its money into cash, when it will earn this rate. The fund can also borrow at the same rate. Draw the feasible set and again say whether a portfolio that is 100% invested in the US is efficient. If the portfolio is not efficient suggest a better portfolio that is efficient.

**In the optimistic case, the efficient portfolios are necessarily some mixture of the minimum variance portfolio (59:41 US:Japan) and cash. In the pessimistic case, need to find the tangency portfolio – the equity portfolio that gives the maximum extra return over the riskless rate per unit of risk. Using solver, we can find this. It has 97% in the US and 3% in Japan – virtually an all US portfolio. This can barely be seen in the graph where the all US portfolio is just below the efficient blue line:**

**The tangency portfolio has volatility of 15.7% and expected return of 9.88%. If we leverage it up by borrowing 2% of the value of portfolio, and putting 102% of the fund into the tangency portfolio, the expected return would only be 10.004% - trivially different from the all US portfolio.**

* 1. The consultants’ report did suggest that the fund put some money into the Japanese market. This suggestion was met with scorn by one trustee, who argued as follows: …

Is the argument valid? (Assume the numbers are correct).

**The case for investing in Japan is not just that it has low correlation with the US, but that it has a low covariance, and hence a low beta, with the US market. The beta of Japan is:**



**To be worth including in the portfolio, the expected return on Japan would need to exceed:**



**which it does. A typical US stock has a beta of one against the US market and would need to be expected to beat the market to be worth including.**